

not observed for smaller values of  $\theta$  and could perhaps be interpreted as being a consequence of local stall. It should finally be noted that the deflection towards the hub agrees with the sudden drop of  $v/\omega R$  observed at  $\theta = 16^\circ$ .

For the variations of  $w/\omega R$  it can be seen in Fig. 4 that a first maximum value is reached at  $\psi \simeq 65^\circ$  for all the values of  $\theta$ . This maximum occurs when the lower side boundary of the blade wake arrives at the probe. This boundary seems to be independent of the pitch blade. The minima observed at  $\psi \simeq 90^\circ$  are due to the blade wake and the subsequent maxima correspond to the upper side boundary of the blade wake. These maxima which occur at larger values of  $\psi$  for larger  $\theta$  are significant evidence of the thickening of the wake produced by the separation of the flow on the blade.

It can be concluded that it is possible to infer some phenomena occurring on the blades by analysing the instantaneous

velocity measured in the near wake of a helicopter rotor. In particular, the sudden change in the profile of  $v/\omega R$  as  $\theta$  increases very likely indicates some local blade stall and this question will be the object of further investigation.

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## Technical Comments

### Comment on "A Graphical Method for the Investigation of Shock Interference Phenomena"

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**K**EEPING abreast of scientific developments in countries other than one's own is becoming more and more difficult. An example of this problem is given by a recent publication in this journal.<sup>1</sup> The graphical method for the straightforward

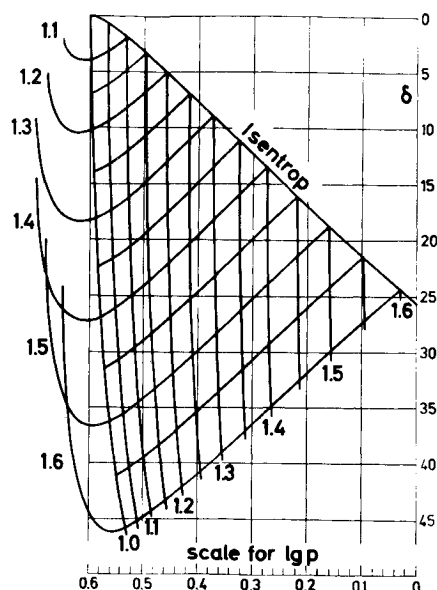


Fig. 1 Herzkurven-diagram after A. Weise, including the isentrop. Laval number  $\omega$  as parameter, the figures on the left margin giving the characterizing values for the different heart-curves drawn.  $\gamma = 1.405$ .

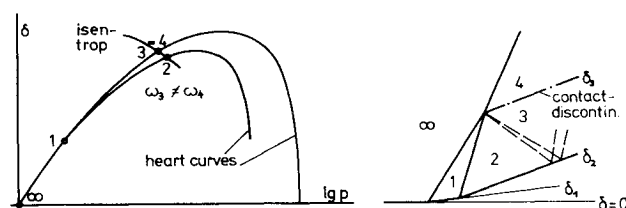


Fig. 2 Flow over double wedge in case of a reflected rarefaction wave.

computation of supersonic flow with imbedded, and eventually interacting, shock waves and contact discontinuities described there by Crawford was developed by Weise<sup>2,5</sup> in 1943–1944 as the "Herzkurven-Methode" (the curves resembling the symbolic heart-shape). As encyclopaedic articles<sup>3,4</sup> confirm, Weise tackled not only the same problems as Crawford, but also some others. He also solved the problem of forked, or  $\lambda$ -, shocks.<sup>5–7</sup> Later the Herzkurven-method was used to calculate and classify the types of supersonic flow through cascades,<sup>8</sup> and its basic idea has been adopted for the graphical computation of unsteady one-dimensional flow.<sup>9,3</sup>

As to the presentation of the heart-curves, it is neither necessary to plot both (symmetric) halves of the curves nor to ratio the pressure  $p$  to a special reference value. The latter is due to the additive feature of the logarithm:  $\log(p/p_2) = \log(p/p_1) + \log(p_1/p_2)$ , and changing the reference pressure results in nothing but a translation of the figure along the pressure axis without changing the shape. Furthermore, it is advantageous to give a scale of the flow velocity along the heart-curve, preferably in form of the Laval number  $\omega$  (velocity divided by critical speed, which is the better alternative to the Mach number because of the constancy of the critical speed in isentropic flow). Each heart-curve is then characterized by the Laval number at its cusp which equals the Laval number ahead of the shock. As a last item, we found it convenient to add the isentrop to the  $\log p$ ,  $\delta$ -diagram ( $\delta$  angle of streamline), likewise with an  $\omega$ -scale, and to line up the heart-curves along it in such a way that the location of any cusp on the isentrop's  $\omega$ -scale corresponds with the characterizing value of that heart-curve. So one set of curves  $\omega = \text{const}$  provides the  $\omega$ -scales for the isentrop and the heart-curves, see Fig. 1. With the help of this universal diagram, the  $\log p$ ,  $\delta$ -plane for a special flow configuration can

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be constructed most easily on transparent paper by simply tracing the appropriate curves. The upper halves, if needed, can be traced on the reverse side of the transparent paper. The isentrop is to be used for all homentropic parts of the flow (as is the Busemann Epicycloid in the  $\omega$ ,  $\delta$ -plane). For instance, in example 2 from Crawford's paper<sup>1</sup> if the merging of the two shocks sheds a reflected rarefaction wave (for  $M_\infty^2 < \{3 + [9 + 16/(\gamma - 1)]^{1/2}\}/4$ , point of inflection of the isentrop) the situation is as sketched in Fig. 2. Though nowadays numerical methods have taken over in the engineering field, the use of graphical methods is still, and will remain, valuable in teaching and analyzing. We fully agree with Crawford in this respect. It is sad to report that Dr. Weise, who was Professor Emeritus from the University of Stuttgart, died on July 25, 1973.

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was concerned primarily with lower Mach number applications and made more frequent use of critical (\*) conditions as reference or normalizing quantities. In the author's work, the shock-polar curves are aligned along the  $y$  axis according to the values of  $M_1$  and  $p/p_{ns}$  at the cusp. Mach number and  $p/p_{ns}$  are the customary nondimensional velocity and pressure values used in hypersonic aerodynamics rather than  $u/a^*$  and  $p/p^*$ . The two symmetric parts of the pressure-deflection curves which correspond to positive and negative deflection are very convenient when complex shock interactions are being studied.

The paper<sup>4</sup> did not intend to present all possible interference calculations which can be made by this method, but rather to illustrate its use with two simple examples. Example 2 is that of two successive shocks; Förster carries these shocks to the point of confluence where a contact discontinuity and an expansion wave occur.<sup>5</sup> Solution to this problem, which is Edney's type-VI interference,<sup>6</sup> can be readily determined by the logarithmic-shock-diagram with extrapolation of the correct heart-shaped curve below the point for the isentropic expansion. Since the distance for extrapolation is so short, it can be accomplished with little uncertainty.

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## Reply by Author to K. M. Förster

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THE author appreciates Förster's<sup>1</sup> pointing out the similarity of earlier work in shock interaction<sup>2,3</sup> to the subject paper<sup>4</sup>; unfortunately, Weise's work has not been widely circulated in this country. The present publication<sup>4</sup> carrying the logarithmic-shock-polar-family to hypersonic Mach numbers essential to analysis of entry problems was the result of an independent study of shock interaction. The earlier work performed during an era of intense research in turbo machinery

## Errata

### Analysis of Bonded Joints in Vehicular Structures

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[AIAA J. 11, 1650-1654 (1973)]

IN Figs. 7 and 9, the normalized normal stress should be labeled  $\sigma_x h/p_o$  rather than  $\sigma_x l/p_o$ , where  $h = h_1 + h_2 + h_3$ .

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